# FS-ADE-WLP-FDTD Method for Calculating Wave Propagation in General Dispersive Materials

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**Within the framework of the finite-difference time-domain (FDTD) and the weighted Laguerre polynomials (WLPs), we derive an effective update equation of the electromagnetic in the dispersive media by introducing the factorization-splitting (FS) schemes and the auxiliary differential equation (ADE). As an example, we employ a 2-D parallel plate waveguide loaded with two dispersive medium columns to calculate the plane wave propagation by using the ADE- FDTD method, the ADE-WLP-FDTD method and the FS-ADE-WLP-FDTD method. Results show that the FS-ADE-WLP-FDTD method is more accurate and effective.**

***Index Terms*—Auxiliary differential equation (ADE), Dispersive media, Weighted Laguerre polynomials, Factorization-splitting scheme**

## I. INTRODUCTION

TO eliminate the Courant-Friedrich-Levy (CFL) stability constraint, some techniques, e. g. the alternating-direction implicit (ADI) [1]-[3] and the locally one dimensional (LOD) [4]-[6], were proposed. Although these techniques can get more accurate simulation results and high computational efficiency than conventional FDTD, a large time step inevitably results in a large numerical dispersion error. Also, an unconditionally stable FDTD method using Laguerre polynomials has been proposed [7]. This marching-on-in-order scheme shows better efficient than the conventional FDTD method when analyzing multi-scale structure.

Based on auxiliary differential equation (ADE), an unconditionally stable WLP-FDTD was proposed to simulate electromagnetic wave propagation in general dispersive materials [8], however, it led to a huge sparse matrix equation, which is very challenging to solve. To solve the huge sparse matrix equation, an efficient algorithm is regularly used to implement the WLP-FDTD method [9], in which the huge sparse matrix equation is solved into a sub-steps procedure with a factorized-splitting scheme.

In this paper, a hybrid algorithm, knows as factorization-splitting ADE-WLP-FDTD, is presented to improve its simulation performance. Compared to the conventional implementation, less CPU runtime is spent. The accuracy and efficiency of the proposed method is verified by simulating electromagnetic wave propagation in a variety of dispersive media.

## II. MATHEMATICAL FORMULATION

With lossless and dispersive media, the Maxwell's equations for the 2-D TE$z$ and auxiliary differential equation can be written as [9]

$$E_\alpha^q(\mathbf{r}) = A_\alpha D_\beta H_z^q(\mathbf{r}) + J_{E\alpha}^q + V_{E\alpha}^{q-1} + V_{S\alpha}^{q-1} \tag{1}$$

$$H_z^q(\mathbf{r}) = b\sum_{\substack{\alpha,\beta \\ \alpha\neq\beta}} D_\alpha E_\beta^q(\mathbf{r}) + V_H^{q-1} \tag{2}$$

with $b = 2/(\mu_0 s)$, $V_H^{q-1} = -2\sum_{k=0,q>0}^{q-1} H_z^k(\mathbf{r})$, $\alpha$, $\beta$ = $x$, $y$, and $\alpha \neq \beta$.

In addition, the auxiliary differential variation $S_{\alpha,1}^q(\mathbf{r})$ along $\alpha$-direction can be read

$$\begin{aligned} S_{\alpha,1}^q(\mathbf{r}) &= \left[a_{\alpha,1}/A_{\alpha,1}\right]E_\alpha^q(\mathbf{r}) - \left[c_{\alpha,1}s/A_{\alpha,1}\right]\sum_{k=0,q>0}^{q-1} S_{\alpha,1}^k(\mathbf{r}) \\ &\quad -\left[d_{\alpha,1}s^2/A_{\alpha,1}\right]\sum_{k=0,q>0}^{q-1}(q-k)S_{\alpha,1}^k(\mathbf{r}) \end{aligned} \tag{3}$$

where $b_{\alpha,1}$, $c_{\alpha,1}$, $d_{\alpha,1}$ are known material constants related to the $\alpha$-component of the electric fields $E_\alpha$, respectively; $s$ is the time-scale factor and $q$ is the order of Laguerre functions; $D_\alpha$ represents the first-order central difference operators along $\alpha$ axes; Moreover, the coefficients $A_\alpha$, $J_{E\alpha}^q$, $V_{E\alpha}^{q-1}$ and $V_{S\alpha}^{q-1}$ in Eq. (1) are given by the following equations, respectively,

$$A_\alpha = A_{\alpha,1}\Big/\left[0.5\varepsilon_0\varepsilon_{\alpha,\infty}s\left(a_{\alpha,1}+A_{\alpha,1}\right)\right] \tag{4}$$

with

$$A_{\alpha,1} = b_{\alpha,1} + 0.5c_{\alpha,1} + 0.25d_{\alpha,1}s^2 .$$

$$J_{E\alpha}^q = -A_{\alpha,1}J_\alpha^q(\mathbf{r})\Big/\left[0.5\varepsilon_0\varepsilon_{\alpha,\infty}s\left(a_{\alpha,1}+A_{\alpha,1}\right)\right], \tag{5}$$

with $J_\alpha^q(\mathbf{r})$ describing the incident electric current excitation source along $\alpha$ axes.

$$V_{E\alpha}^{q-1} = -2A_{\alpha,1}\Big/\left(a_{x,1}+A_{\alpha,1}\right)\sum_{k=0,q>0}^{q-1} E_\alpha^k(\mathbf{r}), \tag{6}$$

$$\begin{aligned} V_{S\alpha}^{q-1} &= (c_{\alpha,1}s - 2A_{\alpha,1})\Big/\left(a_{\alpha,1}+A_{\alpha,1}\right)\sum_{k=0,q>0}^{q-1} S_{\alpha,1}^k(\mathbf{r}) \\ &\quad + d_{\alpha,1}s^2\Big/\left(a_{\alpha,1}+A_{\alpha,1}\right)\sum_{k=0,q>0}^{q-1}(q-k)S_{\alpha,1}^k(\mathbf{r}) \end{aligned}, \tag{7}$$

According to Ref [9] and some manipulations, the implicit formulations of electric fields for the 2-D ADE-WLP-FDTD can be written as

$$\begin{aligned}\left(I-bA_{\alpha}D_{2\beta}\right)E_{\alpha}^{q}&=-A_{\alpha}D_{\beta}V_{H}^{q-1}+V_{E\alpha}^{q-1}\\&-bA_{\alpha}D_{\beta}D_{\alpha}V_{E\beta}^{q-1}+V_{S\beta}^{q-1}+J_{E\alpha}^{q}\end{aligned} \quad (8)$$

where $D_{2\alpha}$ is the second-order central difference operators along $\alpha$ axes. Compared Eq. (8) with Ref [9], one can finds that some parameters determined by dispersive media, $A_{\alpha}$, for example, are included.

## III. Numerical Results

In order to validate the effectiveness of the proposed method, the wave transmission in a 2-D parallel plate waveguide with two dispersive medium columns, shown in Fig.1, is calculated. The numerical calculation is the identical parameters with Ref [8].

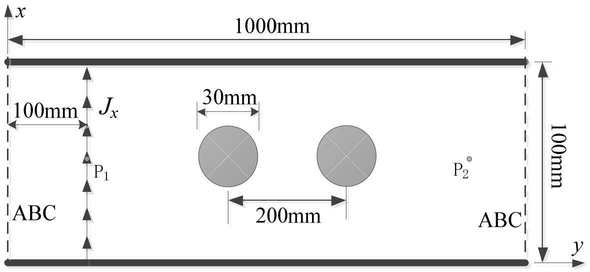


Fig. 1 2-D parallel plate waveguide with two dispersive medium columns.

Fig.2 plots the calculated results given by the FS-ADE-WLP-FDTD, ADE-WLP-FDTD and ADE-FDTD. From the profiles of them, one can find the FS-ADE-WLP-FDTD is accurate.

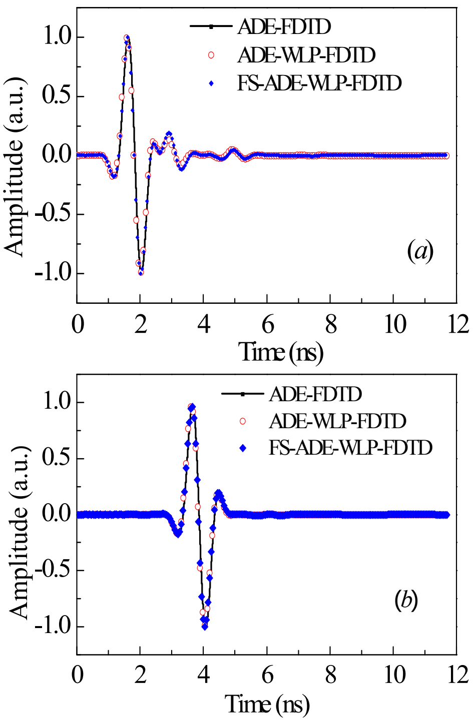


Fig.2. Transient electric fields of the $x$ component (a) at $P_1$ and (b) $P_2$

Table I represents the required computational resource and computing time for the numerical simulations. The proposed method shows much improvement in computation efficiency compared to the ADE-WLP-FDTD and ADE-FDTD.

TABLE I
Comparison of the Computational Efforts for the 2-D Waveguide

| Method | $\Delta t$ (ps) | Meshing size | Marching-on steps | CPU time(s) |
|---|---|---|---|---|
| ADE-FDTD | 0.5 | 320×120 | 23420 | 710 |
| ADE-WLP-FDTD | 30 | 320×120 | 142 | 242 |
| FS-ADE-WLP-FDTD | 30 | 320×120 | 142 | 60 |

## IV. Conclusion

An ADE-WLP-FDTD method based on factorization splitting technique for general dispersive media is presented in this paper. Compared with the ADE-WLP-FDTD, the proposed method can reduce the calculation burden. One numerical example verifies the accuracy and efficiency of the proposed method.

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